section 2

Sheet 2

[1] $\nabla \cdot \vec{f} = 0$

$$\nabla \cdot \vec{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$= -2y\sin(2x+y^2) + 2y\sin(2x+y^2) + 0 = 0$$

Than $\vec{f}$ is Solenoidal

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(5) $F(x,y,z) = x^3yz\,\vec{i} + xy^3z\,\vec{j} + xyz^3\,\vec{k}$

$$\operatorname{curl} \vec{F} = \bar{\nabla}_x \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3yz & xy^2z & xyz^2 \end{vmatrix}$$

$$= (xz^3 - xy^3)\,i - (yz^3 - x^3y)\,\vec{j} + (y^3z - x^3z)\,\vec{k}$$

$$\bar{\nabla} \cdot (\bar{\nabla}_x F) = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$= z^3 - y^3 - z^3 + x^3 + y^3 - x^3 = \text{Zero}$$

Then $(\bar{\nabla}_x F)$ is Solenoidal

[4] $\vec{F} = 2xy^2z\,\vec{i} + 2x^2yz\,\vec{j} + x^2y^2\,\vec{k}$

$$\bar{\nabla}_x \bar{f} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy^2z & 2x^2yz & x^2y^2 \end{vmatrix}$$

$$= (2x^2y - 2x^2y) - (2xy^2 - 2xy^2) + (4xyz - 4xyz) = \vec{0}$$

Then $\vec{F}$ is irrotational

prove $\vec{F} = -\bar{\nabla}\varphi$

electric field → $\vec{F}$; φ → electric potional

$$(f_1\vec{i} + f_2\vec{j} + f_3\vec{k}) = \left(\frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k}\right)$$

$$\varphi_x = -2xy^2z \xrightarrow{\text{integral}} \varphi = -x^2y^2z + A_1(y,z)$$

$$\varphi_y = -2x^2yz \longrightarrow \varphi = -x^2y^2z + A_2(x,z)$$

$$\varphi_z = -x^2y^2 \longrightarrow \varphi = -x^2y^2z + A_3(x,y)$$

$$\varphi = -x^2y^2z$$

$$\therefore \vec{F} = -\bar{\nabla}\varphi$$

DATE

$\boxed{7}$ $\varphi = x^2 + y^2 - 2z^2$ prove $\nabla \cdot (\nabla \varphi) = 0$

$$\vec{\nabla}\varphi = 2x\vec{i} + 2y\vec{j} - 4z\vec{k}$$

$$\vec{\nabla} \cdot (\nabla\varphi) = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} - \frac{\partial f_3}{\partial z}$$

$$= 2 + 2 - 4 = 0$$

$\boxed{6}$ $\varphi = xyz^2$

$$\vec{\nabla}\vec{\varphi} = yz^2\vec{i} + xz^2\vec{j} + 2xyz\vec{k}$$

$$\vec{\nabla} \times (\vec{\nabla}\vec{\varphi}) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz^2 & xz^2 & 2xyz \end{vmatrix}$$

$$= (2xz - 2xz) - (2yz - 2yz) + (z^2 - z^2)$$

$$= \vec{0}$$